Weibull-based Forecasting of R&D Program Budgets

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Presented by

Capt Thomas W. Brown

Tactical Data Links Program

Chief of Cost, ESC/DIV

Hanscom AFB, MA

thomas.brown@hanscom.af.mil

781-377-8052

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Co-Authors:

Maj Edward D. White

Lt Col Mark A. Gallagher

Research Sponsor

The Office of the Secretary of Defense
Program Analysis and Evaluation

Develop an Analytical Model to Phase Cost Estimates for New R&D Program Starts to:

- Assists PA&E in reviewing appropriate R&D program funding and
- Aid Military Departments in forecasting appropriate budget profiles

Overview

- **<u>Background</u>**
- Methodology
- Results
- Conclusion

Background

- Theory: R&D program expenditures are Rayleigh distributed
  - Norden (1970) models manpower utilization
  - Putnam (1978) models software development
  - Watkins (1982) and Abernethy (1984) model defense acquisition data
  - Gallagher and Lee (1996) model to final cost and schedule for ongoing programs
  - Lee, Hogue, and Gallagher (1997) forecast budget profiles from a point estimate

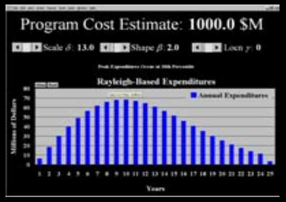
Weibull Function

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}}$$

- The Rayleigh is the Degenerative form of the Weibull
  - Fixed shape parameter, where $\beta = 2$
  - Eliminate the location γ parameter
- Theoretically Limits the Rayleigh

Rayliegh Limitations

- Constant shape parameter ($\beta = 2$)
  - too rigid in predicting the tail portion of expenditures
- No location parameter ($\gamma = 0$)
  - lacks the ability to model the relative program start
- Porter (2001) & Unger (2001) find that Weibull distribution more often supports R&D expenditures



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Weibull Model

$$W(t) = d\left(1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}}\right)$$

t = Time in years

γ = Weibull location parameter (*gamma*)

β = Weibull shape parameter (*beta*)

δ = Weibull scale parameter (*delta*)

d = *cost factor*, where $d = D/.97$*

Lee, Hogue, & Gallagher (1997)

Research Question

Is there a mathematical relationship that can predict the requisite shape and scale parameters to forecast Weibull-based budgets?

Overview

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- **Methodology**
- Results
- Conclusion

Methodology

- Collect & Build Program Model Data
- Multiple Regression Analysis
- Use Lee, Hogue, & Gallagher's (1997) Method of Nonlinear Estimation to Forecast Weibull-based Budgets

Collect & Build Program Model Data

- Data Collection
- Normalize the Data
- Nonlinear Parameter Estimation
- Regression Model Data

Data Collection

- Source: Selective Acquisition Report (SAR)
- Selection Criteria: R&D programs that …
  - were not terminated and
  - had at least 3 budget years to MSIII
- Database consists of 128 R&D programs

Normalize the Data

Budgets to Expenditures

OSD Hypothetical Outlay Rates

S_1	S_2	S_3	S_4	S_5
50%	30%	10%	7%	3%

Fiscal Year	Budget, B_i	Yr-1	Yr-2	Yr-3	Yr-4	Yr-5	Yr-6	Yr-7	Yr-8	Yr-9
2002	50	25.0	15.0	5.0	3.5	1.5				
2003	200		100.0	60.0	20.0	14.0	6.0			
2004	500			250.0	150.0	50.0	35.0	15.0		
2005	150				75.0	45.0	15.0	10.5	4.5	
2006	100					50.0	30.0	10.0	7.0	3.0
Current $ Expenditures		25.0	115.0	315.0	248.5	160.5	86.0	35.5	11.5	3.0
Inflation Index		1.000	1.025	1.050	1.075	1.100	1.125	1.150	1.175	1.200
Constant $ Expenditures		25.0	112.2	300.0	231.2	145.9	76.4	30.9	9.8	2.5

Parameter Estimation

Build our regression response data (Y's)

- Estimate the Weibull β, δ, and γ parameters
- Nonlinear estimation (MS Excel Solver)
- Weibull parameters are the changing cells
- Minimize the $\Sigma(\text{errors})^2$ between the actual cumulative constant dollar expenditures and the Weibull-based cumulative constant dollar expenditures

Model Building Data

Regression Model Data

- Response or dependent variables (Y's)
  - Weibull shape and scale least squares estimates
- Predictors or independent variables (X's)
  - Lead service (Air Force, Navy, Army)
  - Program system type (Aircraft, Electronic, etc.)
  - Total program cost in constant-dollars
  - Total program duration to MSIII in years

Regression Analysis

- Randomly selected 102 (80%) programs to build our shape and scale regression models
- Response (Y's)
  - Least Squares Estimated (LSE) Weibull shape and scale
- Predictors (X's)
  - Cost factor, duration, service branch, and system type
- Test for a mathematical relationship to predict the LSE Weibull shape and scale parameters

Forecast Weibull-based Budgets

- Convert budgets to a total program cost
- Use Lee, Hogue, and Gallagher's (1997) method to forecast Weibull-based budgets from a total program cost
  - convert the total program cost to Weibull-based current-dollar expenditures
  - use MS Excel Solver as our Nonlinear estimation tool
    - target cell minimizes the $\Sigma(\text{errors})^2$ between the Weibull-based current-dollar expenditures and estimated current dollar expenditures
    - changing sells are the year budget dollars

Total Program Cost

- Convert 128 completed budgets to a total program cost, D, with

$$O_i = B_i s_1 + B_{i-1} s_2 + B_{i-2} s_3 + \ldots + B_{i-J} s_J ,$$

$$O^*_i = O_i / c_i \text{, and } D = \Sigma\, O^*_i$$

- Convert the total program cost, D, to a cost factor, d, with $D = E(t_{final}) = 0.97d$*

*Lee, Hogue, and Gallagher (1997)

Model Weibull-Based Expenditures

- Using the regression models to predicted the shape & scale values and applying the cost factor, *d*, we model Weibull-based cumulative constant dollar expenditures, $W(t_i)$, with

$$W(t_i) = d \cdot \left[1 - e^{-\left(\frac{t_i - \gamma}{\delta}\right)^{\beta}} \right]$$

Cumulative Constant $ to Annual Current $

- Convert Weibull-based constant dollar cumulative expenditures $W(t_i)$ to current dollar annual expenditures, $\hat{O}_i$, with

$$O_i = W(t_i) - W(t_{i-1}) \text{ and } \hat{O}_i = O_i c_i$$

Weibull-Based Budgets

- Apply Lee, Hogue, & Gallagher's (1997) nonlinear estimation method to forecast Weibull-based budgets
- Estimate current dollar expenditures, $\tilde{O}_i$, using $\tilde{O}_i = \hat{B}_i s_1 + \hat{B}_{i-1} s_2 + \ldots \hat{B}_{i-J} s_J$, where $\hat{B}_i$ are the changing cells in MS Excel Solver
- Minimize $\Sigma(\text{errors})^2$ between Weibull-based expenditures, $\hat{O}_i$, & estimated current dollar expenditures, $\tilde{O}_i$, using MS Excel Solver with

$$\min \sum_{i=1}^{N} \left(\tilde{O}_i - \hat{O}_i \right)^2$$

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Overview

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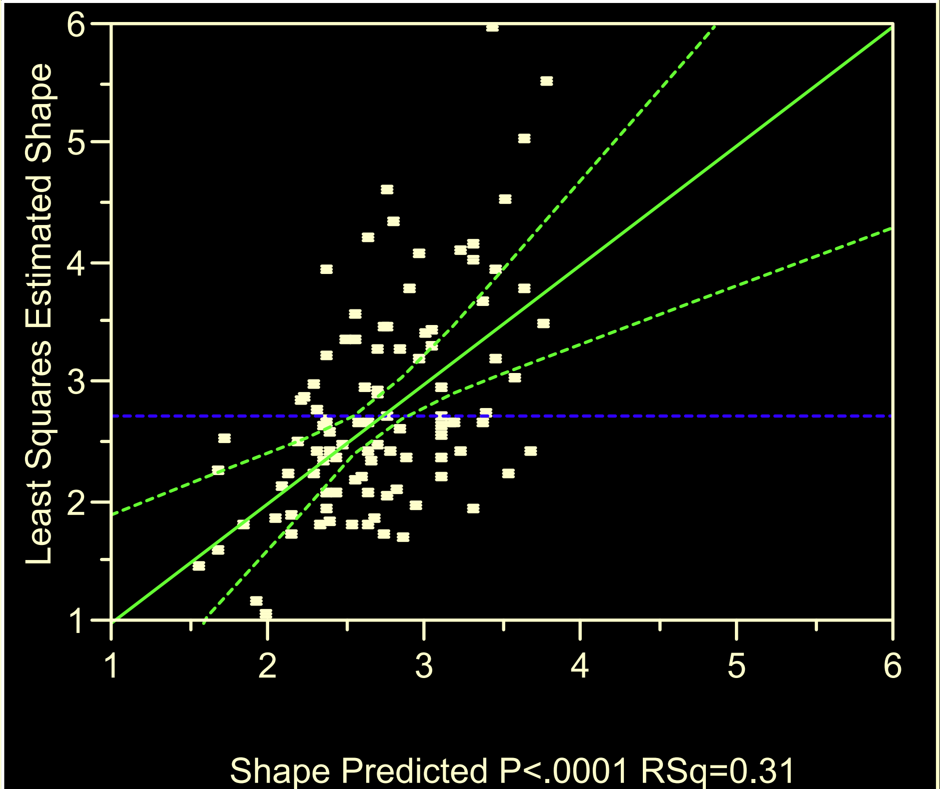
Results

- Shape & Scale Regression Models
- Test Regression Model Assumptions
  - Normality
  - Constant Variance
  - Independence
- Validate Shape & Scale Model Robustness
- Rayleigh & Weibull Model Comparison

Shape β Model

Least Square Estimates by Predicted Plot



Shape Model Summary of Fit	
RSquare	0.310116
RSquare Adj	**0.274185**
Root Mean Square Error	0.763702
Mean of Response	2.724529
Observations (or Sum Wgts)	102

Shape Model Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	25.169127	5.03383	8.6308
Error	96	55.991124	0.58324	**Prob > F**
C. Total	101	81.160251		**<.0001**

Shape Model Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.2995608	0.32514	4	0.0001
ln(1/Dur)	-0.9732540	0.160373	-6.07	<.0001
Army	-0.4234340	0.20643	-2.05	0.043
Navy	-0.4856610	0.188816	-2.57	0.0116
Electronic	-0.5450790	0.181523	-3	0.0034
Space	-1.1001890	0.562901	-1.95	**0.0536**

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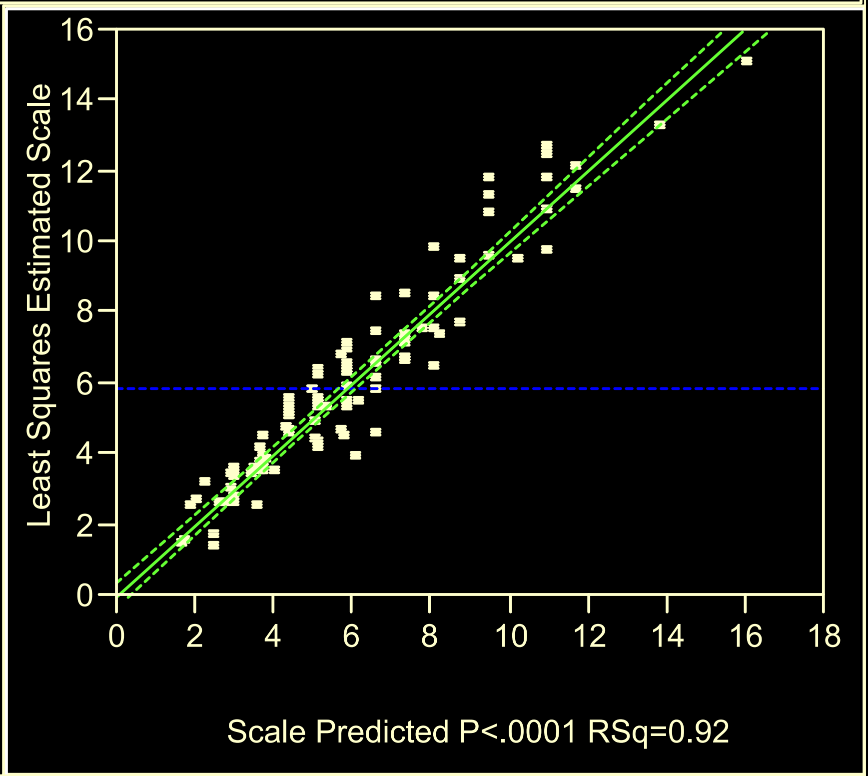
Scale δ Model

Least Square Estimates by Predicted Plot



Scale Model Summary of Fit	
RSquare	0.921671
RSquare Adj	**0.920888**
Root Mean Square Error	0.824422
Mean of Response	5.854373
Observations (or Sum Wgts)	102

Scale Model Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	799.75149	799.751	1176.672
Error	100	67.96724	0.68	Prob > F
C. Total	101	867.71873		**<.0001**

Scale Model Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0683049	0.187391	0.36	0.7163
Duration	0.7256199	0.021153	34.3	**<.0001**

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Final Regression Models

Final Shape Model

Predicted Shape = 1.300 - 0.973(ln(1/Duration)) - 0.423(Army) - 0.486(Navy) - 0.545(Electronics) - 1.100(Space)

Final Scale Model

Predicted Scale = .726(Duration)

Model Validation

Test the Robustness of our regression models

- Did we over-fit the data used to build the models?
  - We determine if the remaining 26 (20%) program LSE shape and scale values fall within a 95% prediction interval
  - 100% and 96% of the LSE ("true") shape and scale values fall within a 95% prediction interval

Conclusion: We did not over-fit the data and both models are robust in predicting the Weibull shape and scale parameters

Rayleigh vs. Weibull

- Use Lee, Hogue, and Gallagher's (1997) method to forecast a budget profile from a point estimates using both the Rayleigh & Weibull Models
- Compare the average correlation between Rayleigh-based & Weibull-based budgets to the 128 completed R&D program budgets

Comparison Results

Correlation Category	Rayleigh	Weibull	Delta
Average Correlation	0.0021	0.6068	0.6047
Minimum Correlation	-0.9051	-0.9984	0.0934
Maximum Correlation	0.9599	0.9986	0.0387

	Correlation Distribution		% Correlation Distribution	
Correlation (c)	Rayleigh	Weibull	Rayleigh	Weibull
Correlation < 0.5	106	37	83%	29%
Correlation ≥ 0.5	22	91	17%	71%

		Correlation < 0.5		% Correlation < 0.5	
Duration	Programs	Rayleigh	Weibull	Rayleigh	Weibull
Duration < 7	51	41	22	80%	43%
Duration ≥ 7	77	66	15	86%	19%

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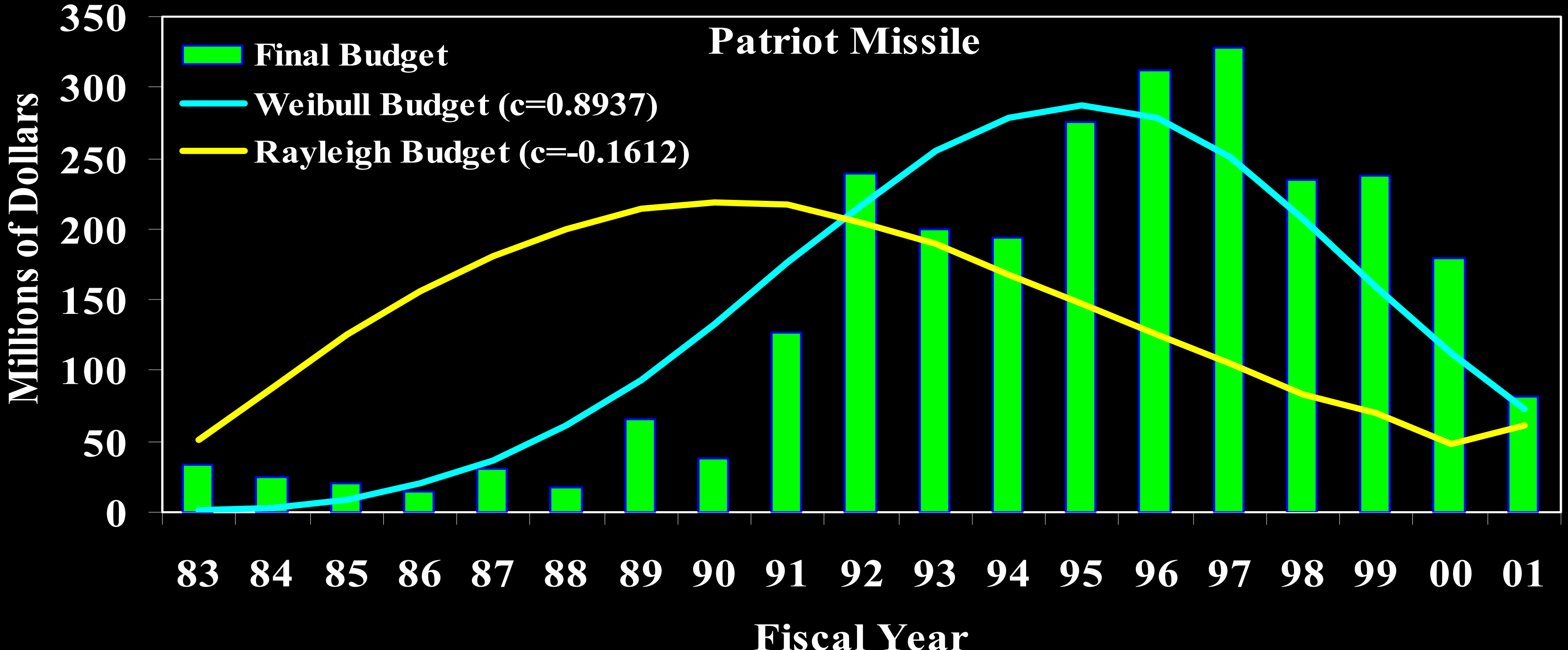
Potentially Misleading

- 52% of Rayleigh-based budgets are negatively correlated (inversely forecasted) to actual budgets



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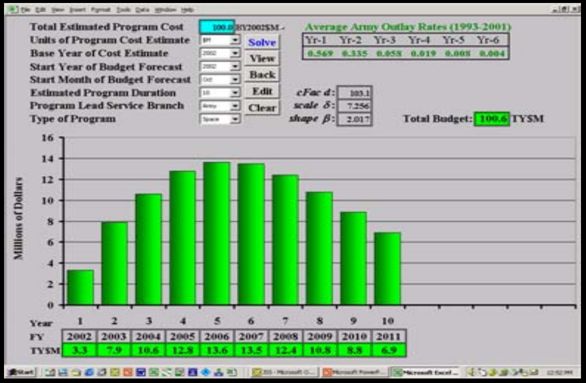
Conclusions

- The Weibull out performs the Rayleigh model when forecasting R&D programs budgets on average 60%
- Potential User Model



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Questions

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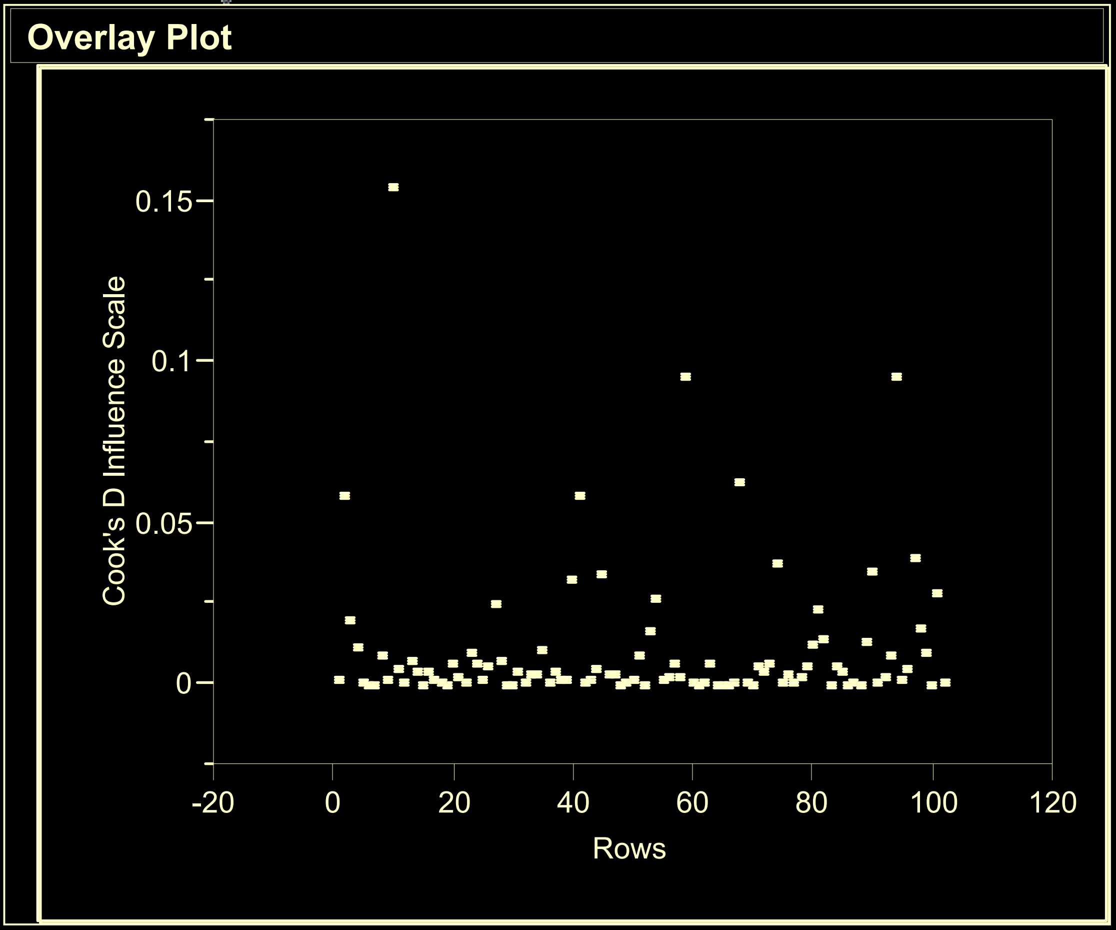
Backup Slides

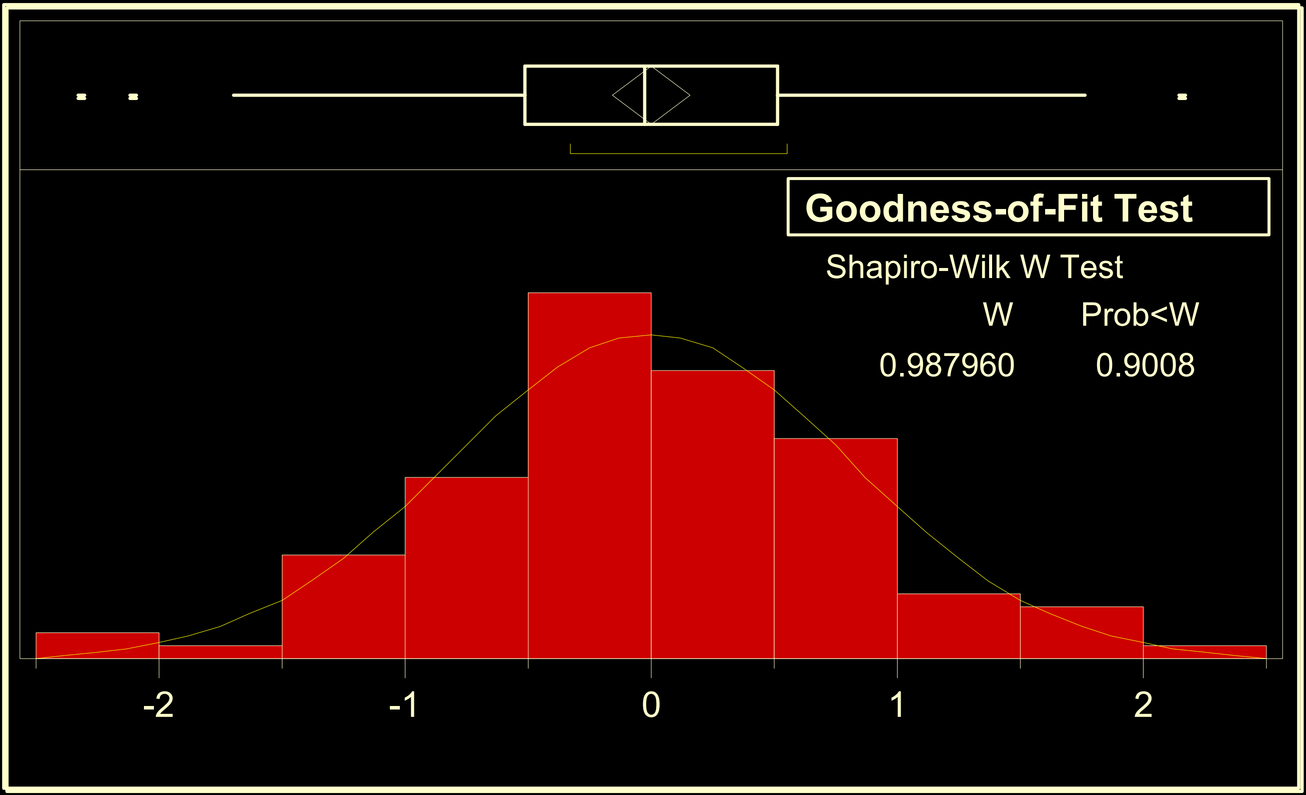
Influential Data Points



- Determines if observations have large effects on our regression parameter estimates.
- Values greater than 0.5 are considered significant influential observations (Neter, 1996)

Scale Model Assumptions

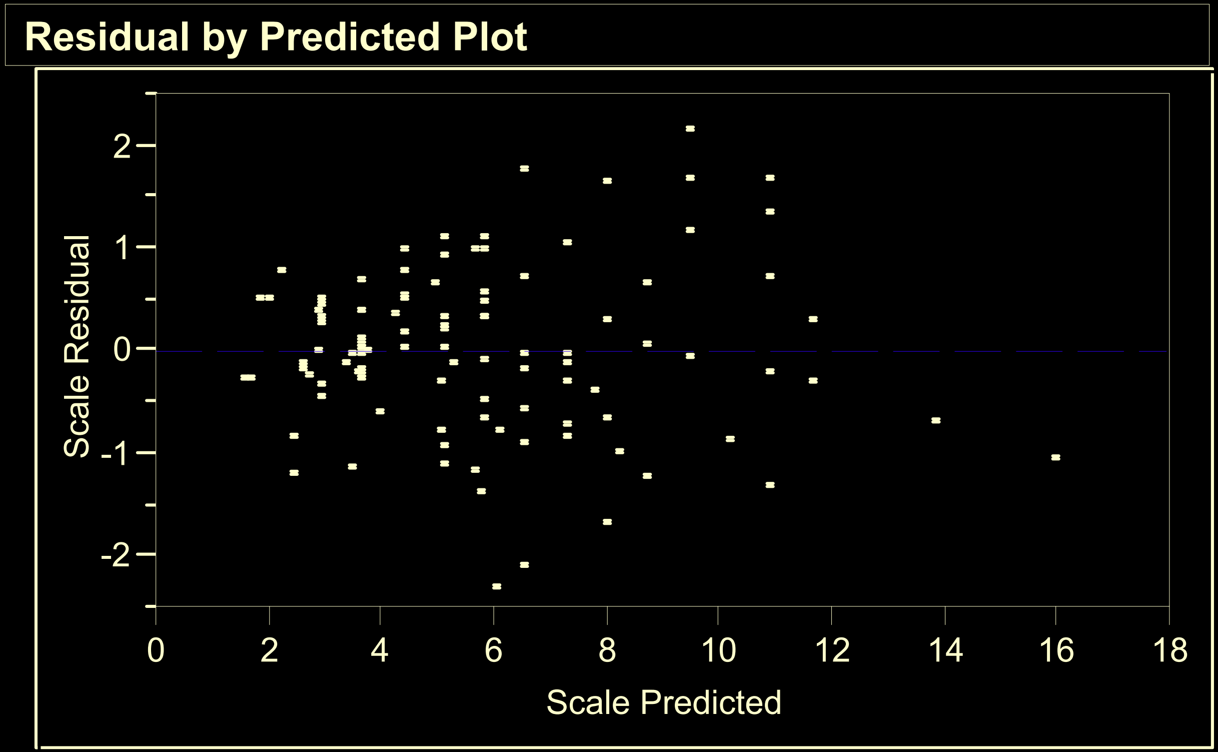
Scale Model Residual Normality Test



- Plot the distribution of the residuals
- Fit a normal curve
- *p* value > 0.05 than residuals are normally distributed

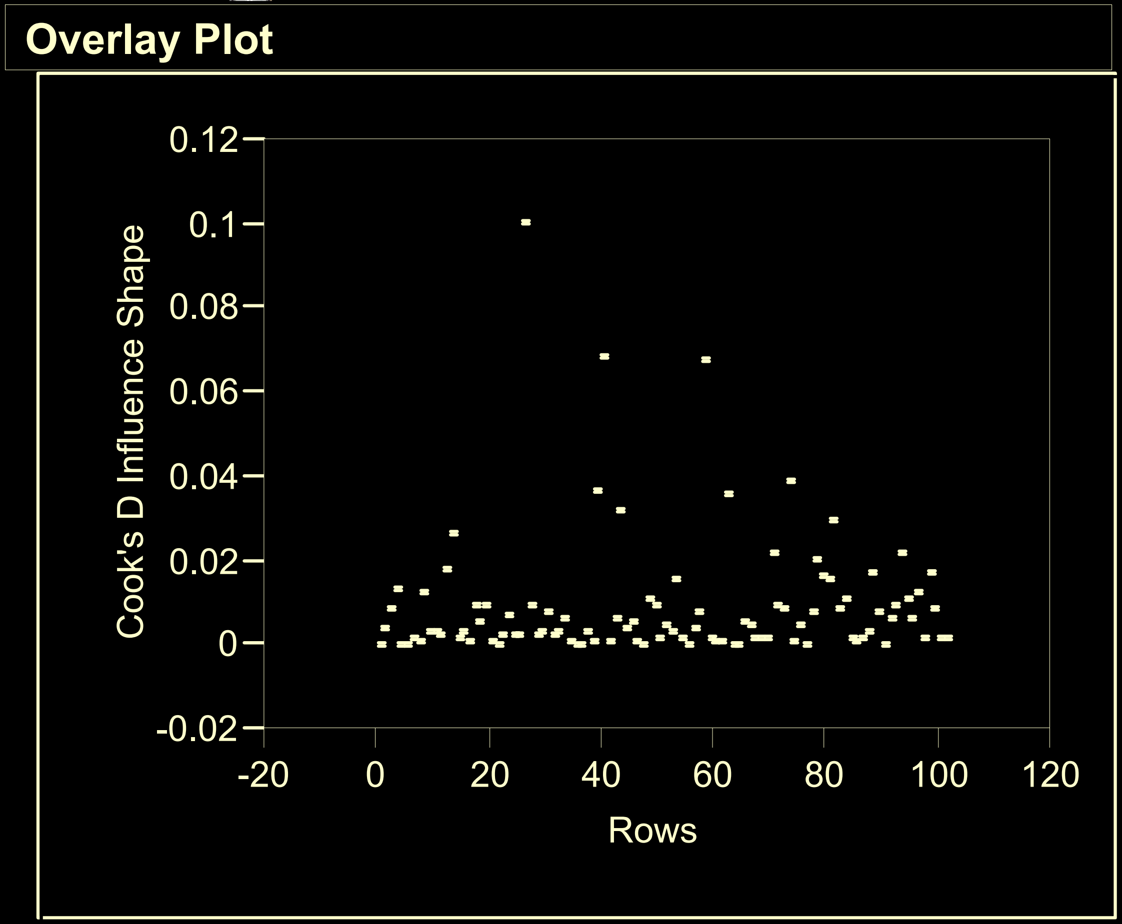
Scale Model Assumptions

Scale Model Constant Variance Test



- Plot the residuals by Predicted
- Visually determine if values are uniformly distributed
- Reasonably uniform distribution

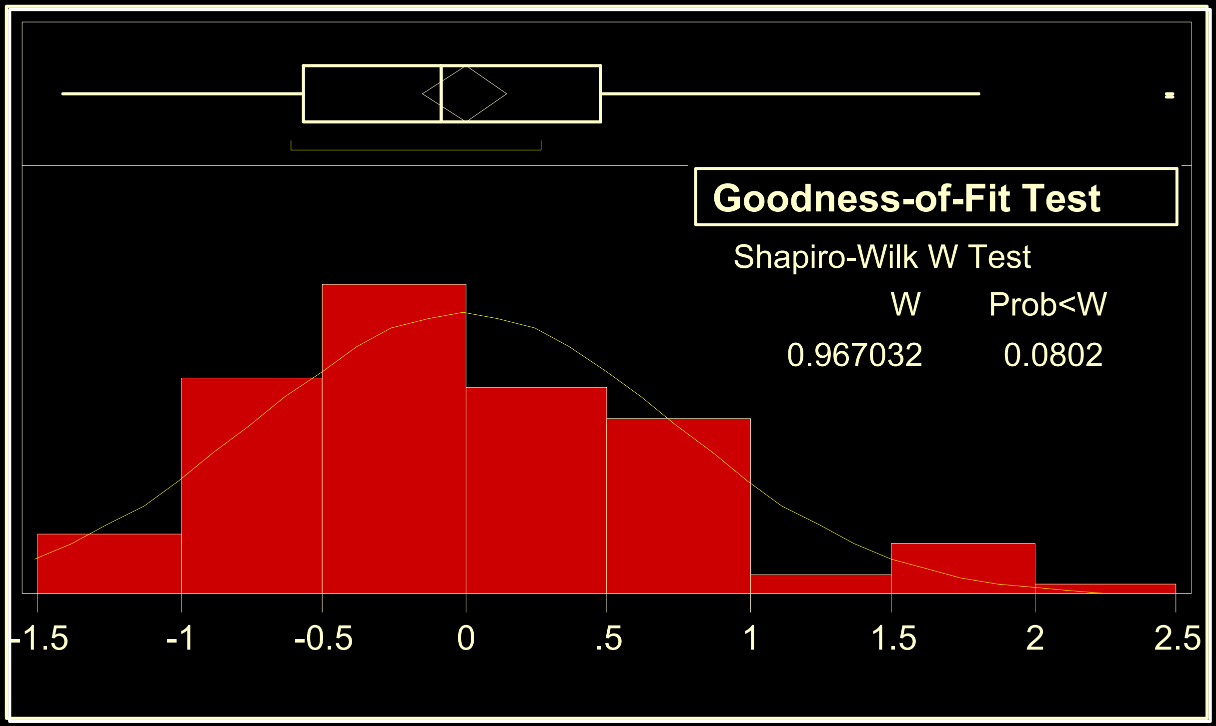
Influential Data Points



- Determines if observations have large effects on our regression parameter estimates.
- Values greater than 0.5 are considered significant influential observations (Neter, 1996)

Shape Model Assumptions

Shape Model Residual Normality Test



- Plot the distribution of the residuals
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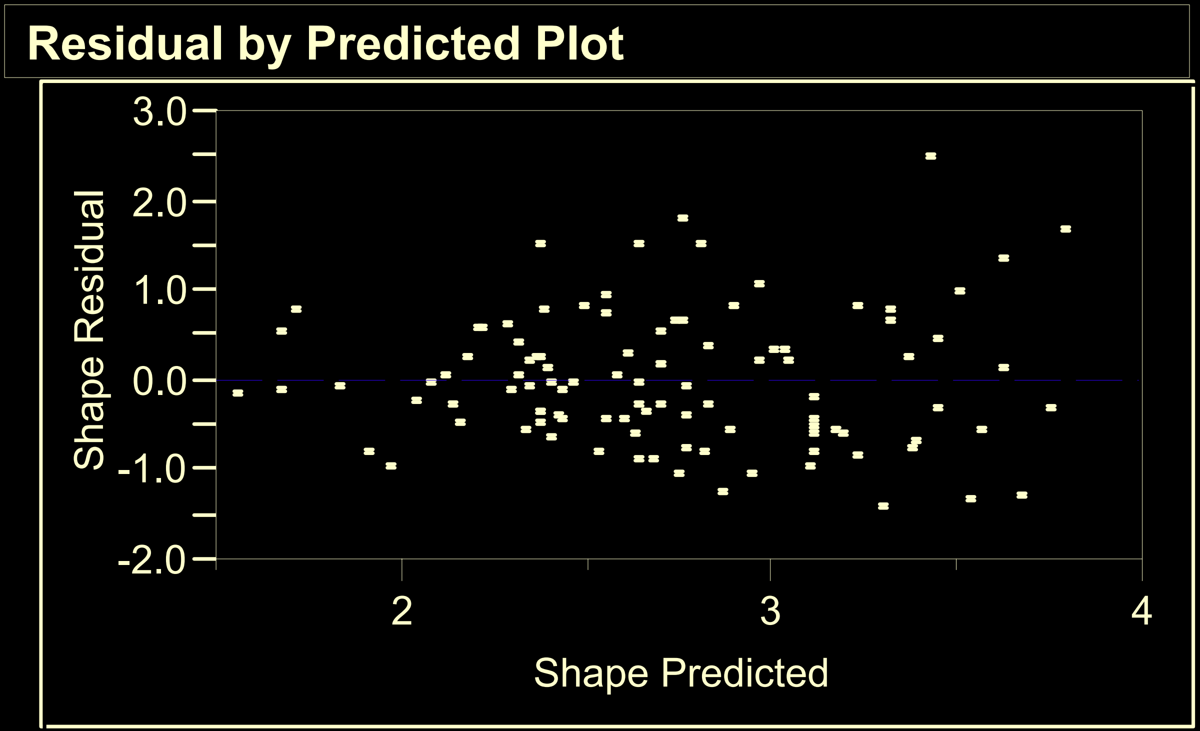
Shape Model Assumptions

Shape Model Constant Variance Test



- Plot the residuals by Predicted
- Visually determine if values are uniformly distributed
- Reasonably uniform distribution

Conclusion

- Limitations
- Future Research
- Conclusion and Questions

Limitations

- Scope of the Research Effort
  - Funding constraints due to budgets not meeting fiscal expenditure requirements
- Accuracy of the Total Program Cost Estimate
- Programs with 4 or less budget years
  - 63 percent are not Weibull distributed
  - Expenditures show no consistent distribution
- Limited to Army, Navy, and Air Force

  ACAT I R&D programs

Future Research

- Compare Initial and Weibull-based forecasted budgets to final budgets
  - Only 13 programs to evaluate
  - Too small to draw any statistical conclusions
- Apply the same methodology to other data sources (lower ACAT programs)

Budgets to Expenditures

OSD Outlay Rates (as Percentages)

s_1	s_2	s_3	s_4	s_5
50%	30%	10%	7%	3%

#	FY	Budget Profile		Expenditure Profile in Current $ Million	
		B_i =	Current$	$O_i = B_i s_1 + B_{i-1} s_2 + B_{1-2} s_3 + \ldots B_{i-J} s_J$	= Current $
1	2002	B_1 =	100.0	$O_1 = B_1 s_1$	= 50.0
2	2003	B_2 =	300.0	$O_2 = B_2 s_1 + B_1 s_2$	= 180.0
3	2004	B_3 =	700.0	$O_3 = B_3 s_1 + B_2 s_2 + B_1 s_3$	= 450.0
4	2005	B_4 =	1100.0	$O_4 = B_4 s_1 + B_3 s_2 + B_2 s_3 + B_1 s_4$	= 797.0
5	2006	B_5 =	1800.0	$O_5 = B_5 s_1 + B_4 s_2 + B_3 s_3 + B_2 s_4 + B_1 s_5$	= 1324.0
6	2007	B_6 =	2500.0	$O_6 = B_6 s_1 + B_5 s_2 + B_4 s_3 + B_3 s_4 + B_2 s_5$	= 1958.0
7	2008	B_7 =	2900.0	$O_7 = B_7 s_1 + B_6 s_2 + B_5 s_3 + B_4 s_4 + B_3 s_5$	= 2478.0
8	2009	B_8 =	300.0	$O_8 = B_8 s_1 + B_7 s_2 + B_6 s_3 + B_5 s_4 + B_4 s_5$	= 1429.0
9	2010	B_9 =	200.0	$O_9 = B_9 s_1 + B_8 s_2 + B_7 s_3 + B_6 s_4 + B_5 s_5$	= 709.0
10	2011	B_{10} =	100.0	$O_{10} = B_{10} s_1 + B_9 s_2 + B_8 s_3 + B_7 s_4 + B_6 s_5$	= 418.0
11	2012			$O_{11} = B_{10} s_2 + B_9 s_3 + B_8 s_4 + B_7 s_5$	= 158.0
12	2013			$O_{12} = B_{10} s_3 + B_9 s_4 + B_8 s_5$	= 33.0
13	2014			$O_{13} = B_{10} s_4 + B_9 s_5$	= 13.0
14	2015			$O_{14} = B_{10} s_5$	= 3.0

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Current $ to Constant $

#	FY	Expenditures Current $	Inflation c_i = Index	Annual Expenditure Profile $O^*_i = O_i / c_i$ = CY02$M
1	2002	O_1 = 50.0	c_1 = 1.0000	$O^*_1 = O_1/c_1$ = 50.0
2	2003	O_2 = 180.0	c_2 = 1.0250	$O^*_2 = O_2/c_2$ = 175.6
3	2004	O_3 = 450.0	c_3 = 1.0500	$O^*_3 = O_3/c_3$ = 428.6
4	2005	O_4 = 797.0	c_4 = 1.0750	$O^*_4 = O_4/c_4$ = 741.4
5	2006	O_5 = 1324.0	c_5 = 1.1000	$O^*_5 = O_5/c_5$ = 1203.6
6	2007	O_6 = 1958.0	c_6 = 1.1250	$O^*_6 = O_6/c_6$ = 1740.4
7	2008	O_7 = 2478.0	c_7 = 1.1500	$O^*_7 = O_7/c_7$ = 2154.8
8	2009	O_8 = 1429.0	c_8 = 1.1750	$O^*_8 = O_8/c_8$ = 1216.2
9	2010	O_9 = 709.0	c_9 = 1.2000	$O^*_9 = O_9/c_9$ = 590.8
10	2011	O_{10} = 418.0	c_{10} = 1.2250	$O^*_{10} = O_{10}/c_{10}$ = 341.2
11	2012	O_{11} = 158.0	c_{11} = 1.2500	$O^*_{11} = O_{11}/c_{11}$ = 126.4
12	2013	O_{12} = 33.0	c_{12} = 1.2750	$O^*_{12} = O_{12}/c_{12}$ = 25.9
13	2014	O_{13} = 13.0	c_{13} = 1.3000	$O^*_{13} = O_{13}/c_{13}$ = 10.0
14	2015	O_{14} = 3.0	c_{14} = 1.3250	$O^*_{14} = O_{14}/c_{14}$ = 2.3

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Perform GOF Statistics

- Perform GOF Statistical Tests Using
  - Komolgorov-Smirnov
  - Cramer-von Mises
  - Anderson-Darling
- Unger (2001) Modifies the Continuous Distribution GOF Tests to Perform GOF Test for Discrete Distributions (Program Expenditures)

Goodness-of-Fit

Komolgorov-Smirnov GOF Results

Program Duration in Years	Programs	Accept	Reject	% Accept	% Reject
Duration<= 3	5	3	2	60%	40%
3<Duration<=4	17	11	6	65%	35%
4<Duration<=5	15	14	1	93%	7%
5<Duration<=6	14	14	0	100%	0%
6<Duration<=7	14	12	2	86%	14%
7<Duration<=22	63	60	3	95%	5%
Total	128	114	14	89%	11%

Goodness-of-Fit

Cramer-von Mises GOF Results

Program Duration in Years	Programs	Accept	Reject	% Accept	% Reject
Duration<= 3	5	0	5	0%	100%
3<Duration<=4	17	1	16	6%	94%
4<Duration<=5	15	7	8	47%	53%
5<Duration<=6	14	10	4	71%	29%
6<Duration<=7	14	13	1	93%	7%
7<Duration<=22	63	60	3	95%	5%
Total	128	91	37	71%	29%

Goodness-of-Fit

Anderson-Darling GOF Results

Program Duration in Years	Programs	Accept	Reject	% Accept	% Reject
Duration<= 3	5	0	5	0%	100%
3<Duration<=4	17	7	10	41%	59%
4<Duration<=5	15	11	4	73%	27%
5<Duration<=6	14	8	6	57%	43%
6<Duration<=7	14	10	4	71%	29%
7<Duration<=22	63	56	7	89%	11%
Total	128	92	36	72%	28%

Goodness-of-Fit

Overall GOF Test Results

Test Type	Accept	Reject	% Accept	% Reject
Kolmogorov-Simerof (K-S)	114	14	89%	11%
Cramer-von Mises (CvM)	91	37	71%	29%
Anderson-Darling (A-D)	92	36	72%	28%
Total	297	87	**77%**	23%

Goodness-of-Fit

GOF Results for Budgets ≤ 6 Years

Test Type (51 Programs)	Accept	Reject	% Accept	% Reject
Kolmogorov-Simerof (K-S)	42	9	82%	18%
Cramer-von Mises (CvM)	18	33	35%	65%
Anderson-Darling (A-D)	26	25	51%	49%
Total	86	67	**56%**	44%

GOF Results for Budgets > 6 Years

Test Type (77 Programs)	Accept	Reject	% Accept	% Reject
Kolmogorov-Simerof (K-S)	72	5	94%	6%
Cramer-von Mises (CvM)	73	4	95%	5%
Anderson-Darling (A-D)	66	11	86%	14%
Total	211	20	**91%**	9%

Regression Analysis

- Test for a relationship between the least squares estimated Weibull scale and shape parameters and possible predictors

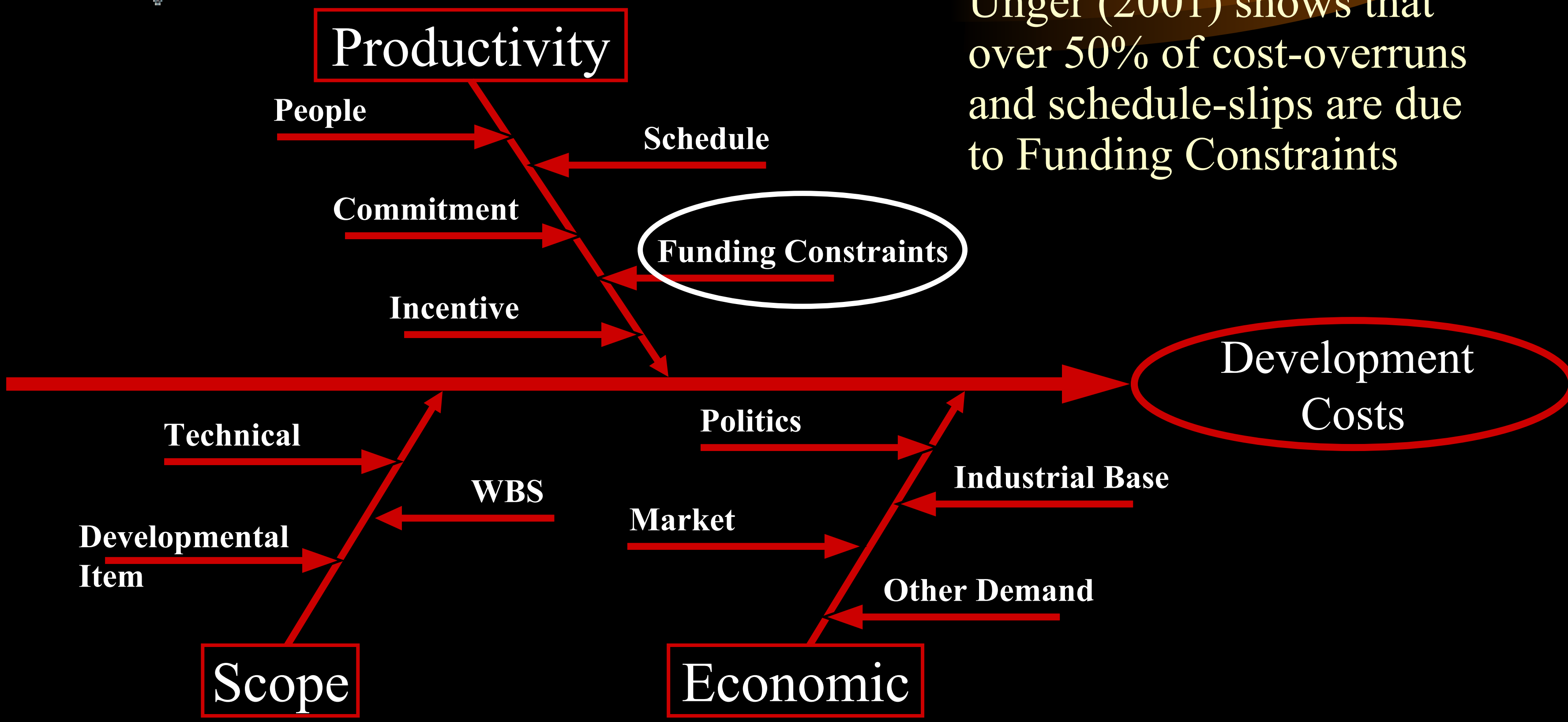
Continuous	Nominal	Nominal	Nominal	Nominal
X_1=Cost	X_3=Army	X_5=Aircraft	X_7=Missile	X_9=Ship
X_2=Duration	X_4=Navy	X_6=Electronic	X_8=Munitions	X_{10}=Space

$$shape(\hat{\beta}) = \beta_0 + \beta_1(X_1) + \beta_2(X_2) + \ldots \beta_i(X_i)$$

$$scale(\hat{\delta}) = \beta_0 + \beta_1(X_1) + \beta_2(X_2) + \ldots \beta_i(X_i)$$

Cost Contributors

Unger (2001) shows that over 50% of cost-overruns and schedule-slips are due to Funding Constraints



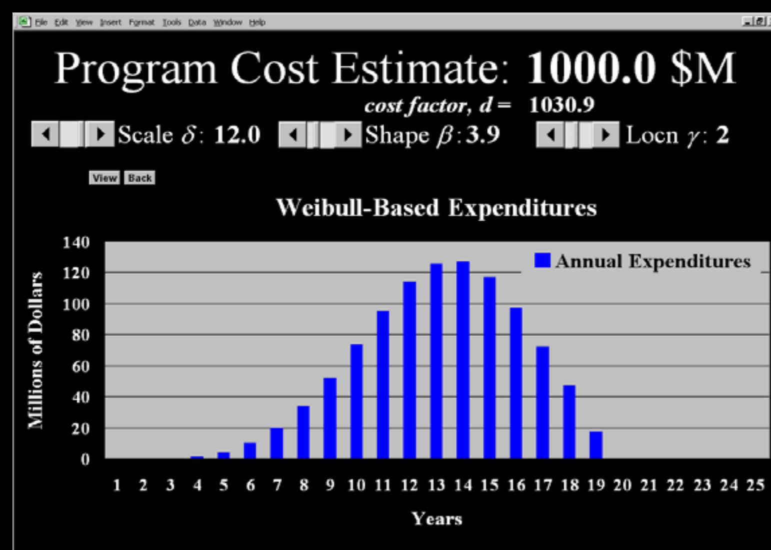
Source: Belcher & Dukovich (2000)

Shape & Scale Model

- Tight fit of LSE scale values to our predicted scale regression line
- Indicating that our scale model predicts scale well
- Adjusted R Square—Compares across models with different numbers of parameters using the degrees of freedom in the computation
- Penalizes models for predictors that may increase the R Square but are statistically insignificant (Over-fitting the data)

Weibull Model Flexibility

- Models insignificant funding
- Shape parameter varies giving flexibility in modeling the tail of expenditures



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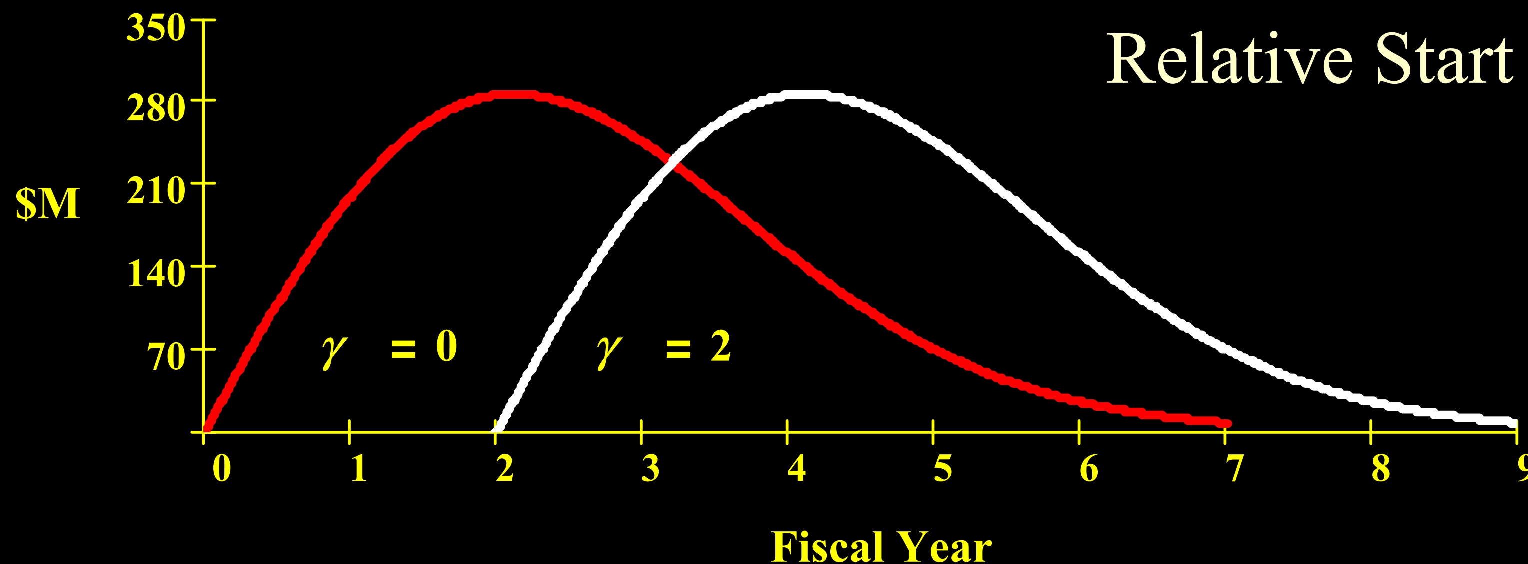
Location (γ) Parameter

$$F(t) = d\left[1 - e^{-\left(\frac{time - location}{scale}\right)^{shape}}\right]$$



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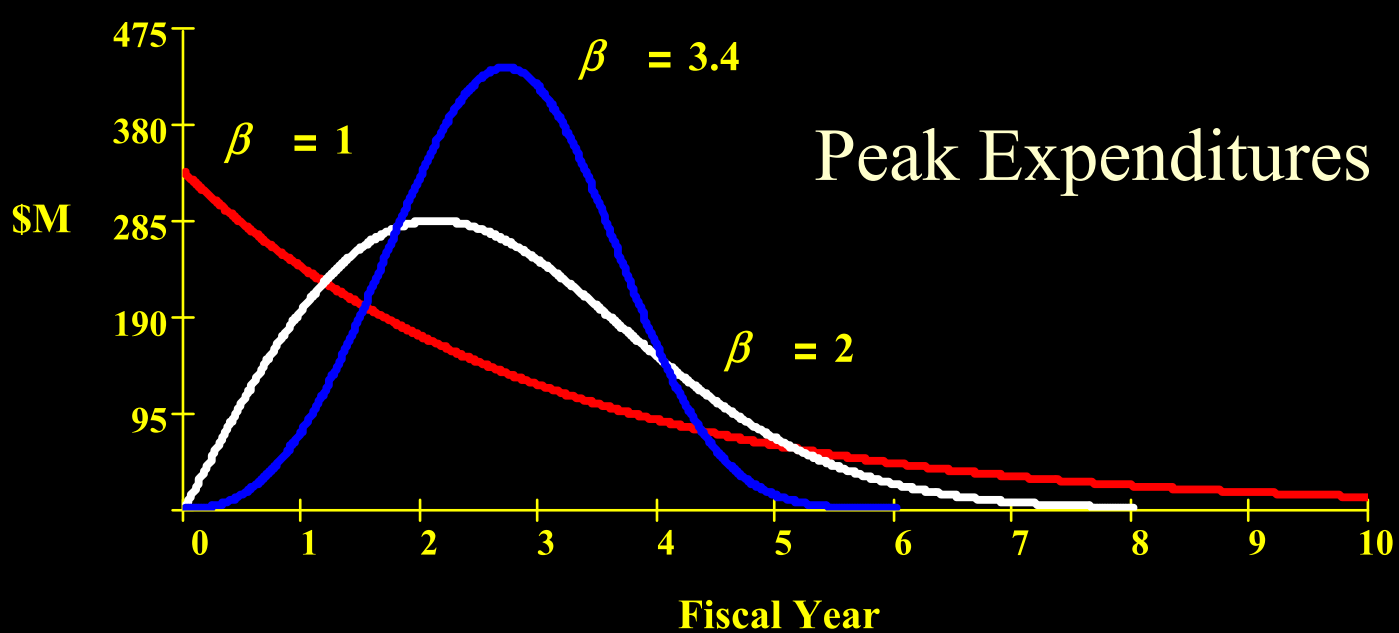
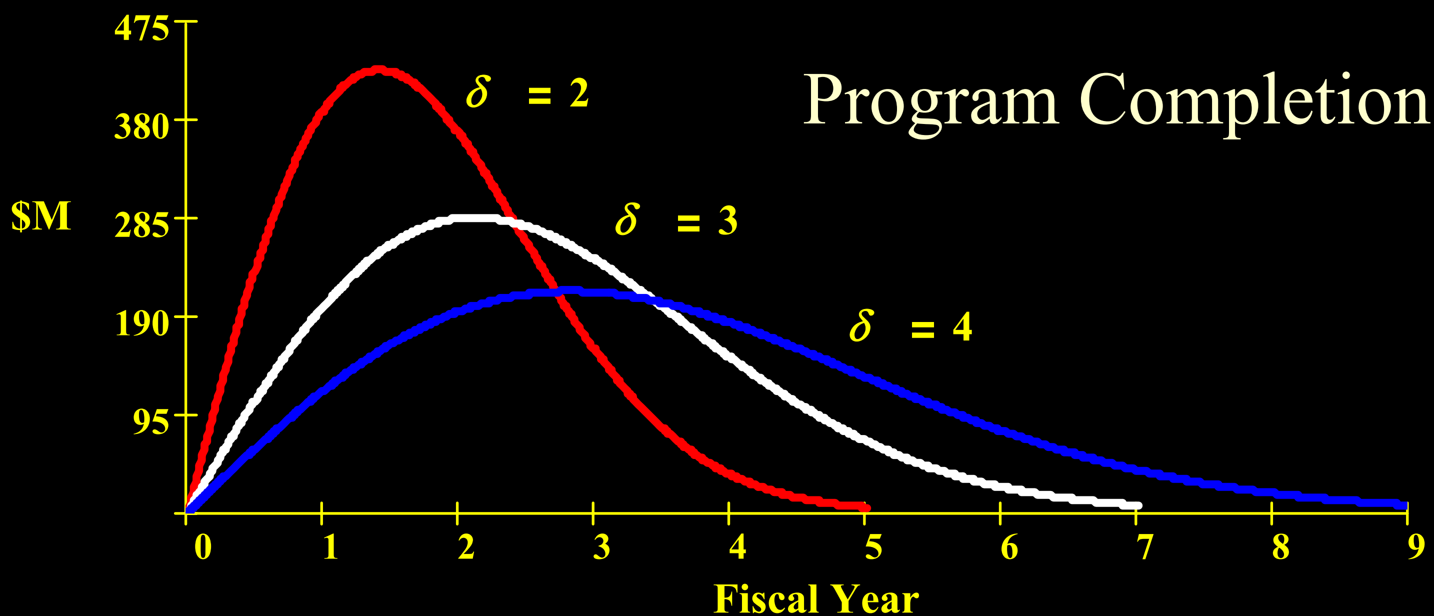
Shape (β) Parameter

$$F(t) = d\left[1 - e^{-\left(\frac{time\ -\ location}{scale}\right)^{shape}}\right]$$



Scale (δ) Parameter

$$F(t) = d\left[1 - e^{-\left(\frac{time\ -\ location}{scale}\right)^{shape}}\right]$$



AFIT

Budgets to Expenditures

- Total Obligation Authority (TOA)
  - Budget profile (B_i) in current (Then Year) dollars
- Outlay rates determine amount spent (s_J)
- Expenditure profile in current dollars (O_i)

$$O_i = B_i s_1 + B_{i-1} s_2 + B_{i-2} s_3 + \ldots + B_{i-J} s_J$$

  - O_i yearly current dollar expenditures
  - B_i yearly budget dollars
  - s_J yearly average outlay rates

Current $ to Constant $

- Expenditures are in current dollars
  - Current dollars have inflation factor
- Remove inflation factor

$$O^*_i = O_i / c_i$$

  - O^*_i yearly constant dollar expenditures
  - O_i yearly current dollar expenditures
  - c_i inflation indices

Purpose

- Who? OSD PA&E & Military Departments
- What? Analytical tool to forecast R&D budget profiles
- When? New R&D program starts
- Why? Determine reasonableness & improve forecasting of R&D program budget profiles
- How? Weibull Model
- Research Question: Is there a mathematical relationship that can predict the requisite shape and scale parameters to forecast Weibull-based budgets?